# Round 1 Questions

16 students, 4 groups of 4, first 2 to one integral (2 minutes )

1. $$\int \frac{1}{\sqrt{x+1}+\sqrt{x}}dx$$

   Mutliply by conjugate!

   Solution:$\frac{2}{3}(-x^{3/2}+(1+x)^{3/2})+C$

2. $$\int_1^3 1-\sqrt{1-(x-2)^2}\,dx$$

   answer: By converting to an area problem $2-\frac{\pi}{2}$

3. $$\int_3^4 x\sqrt{x-3}\,dx$$

   answer $=\frac{12}{5}$

   solution : (substitute $u=x-3$ noting that $x=u+3$ )

4. $$\int_0^2 \frac{x^3}{\sqrt{x^2+1}}\,dx$$

   Answer: $\frac{2}{3}(1+\sqrt{5})$

   Method: Substitute $u=x^2+1$ and rewrite $x^2=u-1$.

5. $$\int \frac{1}{x^2+4x+8}\,dx.$$

   **Solution:** $\dfrac{1}{2}\arctan\left(\dfrac{x+2}{2}\right)+C$

6. $$\int_0^{\pi^2} \cos\sqrt{x}\,dx$$

   Answer: $-4$

   Method: Substitution and integration by parts.

7.

$$\int \frac{1}{x^3 + x} \mathbf{dx}$$

(partial fractions) **answer:** $\ln|x| - \frac{1}{2}\ln|1 + x^2| + C$

8.

$$\int \sqrt{4 + (e^{2x} - e^{-2x})^2} dx$$

solution : $e^{2x}/2 - e^{-2x}/2.$

# Round 2 Questions

8 students, 2 groups of 4, first 2 to one integral in each group (2 minutes for each integral )

1. $$\int \frac{1+2x^{2023}}{x+x^{2024}}\,dx$$

   Answer : $\ln|x| + \frac{1}{2023}\ln|1+x^{2023}| + C$

   Decompose and substitute!

2. $$\int e^{\cos t}\sin(2t)dt$$

   Answer: $-2e^{\cos t}(\cos t - 1) \quad + C$

   Double angle formula+substitution+ by parts

3. $$\int\limits_0^1 x^3\sqrt{1-x^2}\,dx$$

   $\frac{2}{15}$

   (direct or trigonometric substitution(s) can be used )

4. $$\int_0^{\pi} \frac{\cos^3 x}{1+\sin^2 x}\,dx$$

   Answer: 0

   Method: Symmetry around $\pi/2$. Let $u = x - \pi/2$, then use odd function

5. $$\int_0^1 \frac{xe^x}{(1+x)^2}\,dx$$

   answer: $\dfrac{e-2}{2}$

   solution : (integration by parts $u = xe^x, dv = \frac{1}{(1+x)^2}$ )

6. $$\int\limits_{-5}^{5} |2-|1-|x|||dx$$

   answer : 11

   absolute value breaking down

7. Evaluate

$$\int \frac{\sin(2x)}{1+\cos^4 x}dx$$

answer: $-\tan^{-1}(\cos^2 x) + C$

double angle+substitution

8. Evaluate

$$\int \sec^3 x\, dx$$

answer: $\frac{1}{2}\tan(x)\sec(x) + \frac{1}{2}\ln|\sec x + \tan x| + C$

# Round 3 Semi- Finals Questions

4 students, 2 pairs, first one to two integrals in each pair (3 minutes for each integral )

1.
$$\int_1^2 \frac{dx}{x^2\sqrt[4]{(1+x^4)^3}}.$$

Answer :
$$\sqrt[4]{2}-\frac{1}{2}\sqrt[4]{17}$$

Solution : Observe that $(1+x^4)^3 = x^3(1+x^{-4})^3$, then
$$\int \frac{dx}{x^2\sqrt[4]{(1+x^4)^3}} = \int \frac{x^{-5}dx}{\sqrt[4]{(1+x^{-4})^3}}.$$

Next, use the substitution $u = 1+x^{-4}$ to end up with
$$-\frac{1}{4}\int u^{-\frac{3}{4}}du = -\sqrt[4]{1+x^{-4}}$$

2.
$$\int \frac{x^8}{1+x^2}\,dx$$

answer:
$$\frac{x^7}{7}-\frac{x^5}{5}+\frac{x^3}{3}-x+\arctan(x)+C$$

(long divide and then integrate term-by-term )

3.
$$\int_0^\pi \frac{\sin 3t}{\sin t}\,dt$$

Answer:
$$\pi$$

Method: Need trig identity $\sin 3t = \sin t(2\cos 2t+1)$

4.
$$\int e^x\sqrt{9-e^{2x}}\,dx$$

answer :
$$\frac{9}{2}\arcsin\left(\frac{e^x}{3}\right)+\frac{e^x}{2}\sqrt{9-e^{2x}}+C$$

Sub +Trig substitution : $e^x = 3\sin\theta$

5.

$$\int \frac{1}{(1+x^2)^2}\,dx$$

answer:

$$\frac{1}{2}\arctan x + \frac{x}{2(1+x^2)} + C$$

(substitute $x = \tan\theta$ medium / hard)

6.

$$\int\limits_0^{2\pi} \cos(x)\cos(2x)\cos(3x)dx$$

Answer =

$$\frac{\pi}{2}$$

$\frac{x}{4} + \frac{1}{8}\sin(2x) + \frac{1}{16}\sin(4x) \ + \frac{1}{24}\sin(6x)$

7.

$$\int \frac{\ln(ex)}{\sqrt{x\ln(x)}}\,dx$$

Solution :

$$2\sqrt{x\ln(x)} + C$$

Use the following change of variable: $u = x\ln(x) \Rightarrow du = (\ln(x)+1)dx$, thus

$$\int \frac{\ln(ex)}{\sqrt{x\ln(x)}}\,dx = 2\int \frac{du}{2\sqrt{u}} = 2\sqrt{u} = 2\sqrt{x\ln(x)} + C.$$

8.

$$\int \frac{1}{x\sqrt{x^{2024}-1}}dx$$

Solution :

$$\frac{1}{1012}\tan^{-1}(\sqrt{x^{2024}-1}) + C$$

u-substitution then rationalization substitution

9.

$$\int \sqrt{4+(e^{2x}-e^{-2x})^2}dx$$

solution :

$$e^{2x}/2 - e^{-2x}/2$$

.

distribute the square simplify and write as a square again to get rid of square root.

# Round 4 : FINALS!! Questions

**2 students compete for third versus 4th place**
**2 students compete for first versus second place**
**First one to solve two integrals (4minutes each)**

- 1.
$$\int_0^{\pi} \frac{x\sin x}{1+\sin x} dx$$
Answer: $\frac{\pi(\pi-2)}{2}$
Difficulty: Difficult
Method: Can multiply by conjugate +integration by parts or : Using symmetry to write $\int_0^{\pi} x f(\sin x) dx = \pi \int_0^{\pi/2} f(\sin x) dx$ and half-angle substitution

2.
$$\int \frac{1}{\sqrt{x+2}+\sqrt[3]{x+2}}\, dx$$
Answer : $6(2+x)^{1/6} - 3(2+x)^{1/3} + 2\sqrt{2+x} - 6ln|1+(2+x)^{1/6}|$

Solution : let $z = \sqrt[6]{x+2} \Rightarrow z^6 = x+2 \Rightarrow 6z^5\ dz = dx,\ \sqrt{x+2} = z^3,\ \sqrt[3]{x+2} = z^2$, then long division.

3. $\int \sin(101x)\sin^{99} x dx$
Answer : $\frac{\sin(100x)\sin^{100} x}{100} + C$
trig identities $\sin(100x+x) = \sin(100x)\cos x \sin^{99} x + \cos(100x)\sin x \sin^{99} x$ , distribute, integration by parts for $\int \sin(100x)\cos x \sin^{99} x$ leads to a term cancellation.

4. $\int \sqrt{x\sqrt{x\sqrt{x\sqrt{x\sqrt{x\sqrt{x\sqrt{\ldots}}}}}}} dx$
solution : $\int x^{1/2+1/4+1/8+1/16+\ldots} dx = \displaystyle\int x dx = \frac{1}{2}x^2 + C$