Carnegie Mellon University Qatar

Pi Day Math Competition

Final Round 2024

Question 1

The degree of the polynomial $P(x) = (x^2 + 5)^n (x^3 + 7x - 2)^5$ is 27.

What is the value of n?

Question 2

It is given that $\sqrt[n]{\dfrac{12^3 + 12^3 + 12^3}{3^4 + 3^4 + 3^4 + 3^4}} = 2.$

What is the value of n?

Question 3

Two perpendicular lines pass through $(2, 2)$. If one of the lines passes through $(6, 0)$ also, what is the x-intercept of the other line?

Question 4

An object is launched from the ground and reaches a maximum height of 100 meters in 5 seconds after launch and returns to the ground after another 5 seconds. A quadratic function can be used to model the height, in meters, of the object above the ground in terms of time, in seconds after launch.

Based on this model, what is the height of the object 2.5 seconds after launch?

Question 5

What is the sum of the first 100 positive odd integers?

Question 6

Let a, b and c be positive integers such that $\frac{ab}{4} = \frac{ac}{8} = \frac{bc}{12}$ and $2a - 3b + 4c = 57$.

What is the value of b?

Question 7

What is the area of the largest triangle that can be inscribed in a circle of radius 2?

Question 8

It is given that the roots of the quadratic equation $x^2 - (s-3)x + 5t = 0$ are s and t.

What is the sum of s and t?

Question 9

Let abc and cba be 3$-$digit integers such that $abc - cba = 396$.

How many different triples a, b, c satisfy this property?

Question 10

What is the number of consecutive 9's at the right end of $34! - 1$ once it is written in the decimal base?

Question 11

Let $(2x - 3)^2 + (y + 4)^2 = 0$ and $4ax + 5y + 2 = 0$.

What is the value of a?

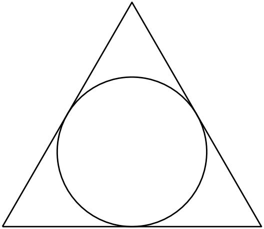
Question 12

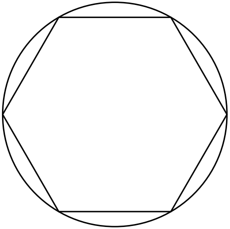
Let $A = 4! + 5!$ and $B = 5! + 6!$.

What is the greatest common divisor of A and B?

Question 13

In the following diagram, there are two circles. The first circle is inscribed in an equilateral triangle. The second circle circumscribes a regular hexagon. If the perimeters of the triangle and hexagon are equal, what is the ratio of the area of second circle to the area of the first circle?





Question 14

Let x and y be positive integers such that $5 < x < 9$ and $2 < y < 6$.

What is the value of $\frac{7x-2y}{y}$ provided that it has an integer value?

Question 15

We know that $2024 = 2 \times 2 \times 2 \times 11 \times 23$.

In how many different ways can we write 2024 as a product of 3 positive integers?

In other words, how many solutions are there to $abc = 2024$ where a, b, c are integers and $1 \leq a \leq b \leq c$?

Question 16

A sequence of numbers $(a_1, a_2, a_3, \ldots)$ is defined recursively as follows: $a_1 = 1$, $a_2 = 2$ and $a_n = \dfrac{a_{n-1} + 1}{a_{n-2}}$ if $n > 2$.

What is the value of a_{100}?