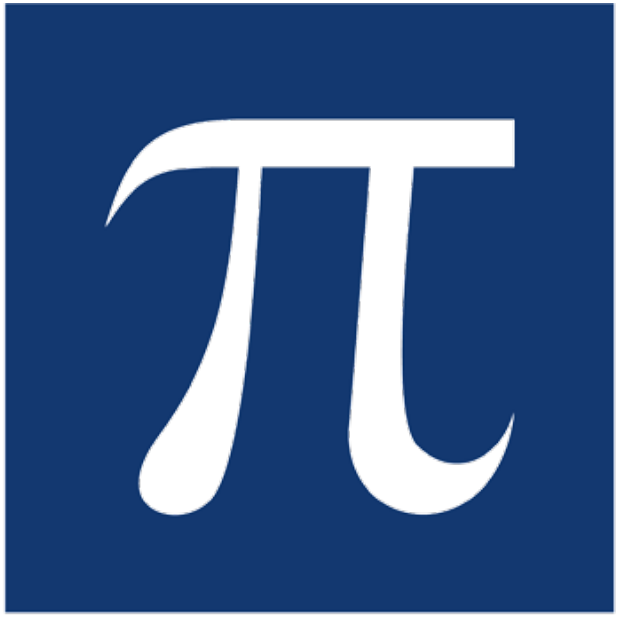
Carnegie Mellon University Qatar



Pi Day Math Competition

Final Round 2023

Question 1

What is the value of

$$100^2 - 99^2 + 98^2 - 97^2 + \cdots + 2^2 - 1^2?$$

Question 2

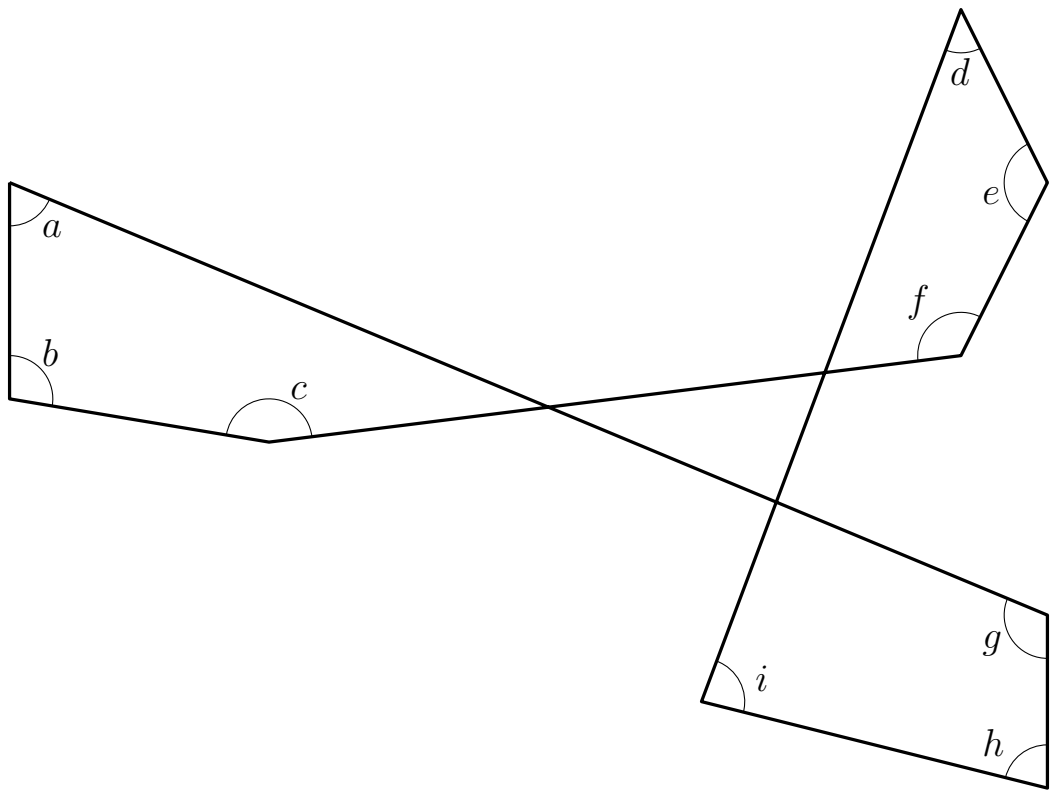
How many three digit positive integers are there for which the sum of digits is odd?

Question 3

How many positive integers are multiple of either 3 or 5 and are strictly less than 1000?

Question 4

What is the sum of the labeled angles $a, b, \ldots, i$ in the figure below?



Question 5

Let a, b, c be positive real numbers such that arithmetic mean of a and b is 3, arithmetic mean of a and c is 4, and geometric mean of b and c is 2.

What is the value of the expression $\frac{c}{b} + \frac{b}{c}$?

Question 6

Suppose $\sin(3x) = \cos(x - 10^\circ)$ where $3x$ and $x - 10^\circ$ are both acute angles.

What is the value of x?

Question 7

What is the value of

$$\log_5 7 \cdot \log_7 9 \cdot \log_9 11 \cdot \cdots \cdot \log_{21} 23 \cdot \log_{23} 25\,?$$

Question 8

Your mathematics teacher replaces the usual numbers on a fair 6-sided die with $2, 3, 5, 8, 13$ and 21. The die is then rolled twice and the numbers are added together. What is the probability that the resulting sum is also a value on the die?

Question 9

What is the last digit in the decimal representation of $2023^{3^{14}}$?

Question 10

A cube has an internal point P such that the perpendicular distances from P to the six faces of the cube are 1 cm, 2 cm, 3 cm, 4 cm, 5 cm and 6 cm. How many *other* internal points of the cube have this property?

Question 11

What is the sum of the roots/solutions of the equation

$$|x+1|^2 - 5|x+1| + 6 = 0\,?$$

Question 12

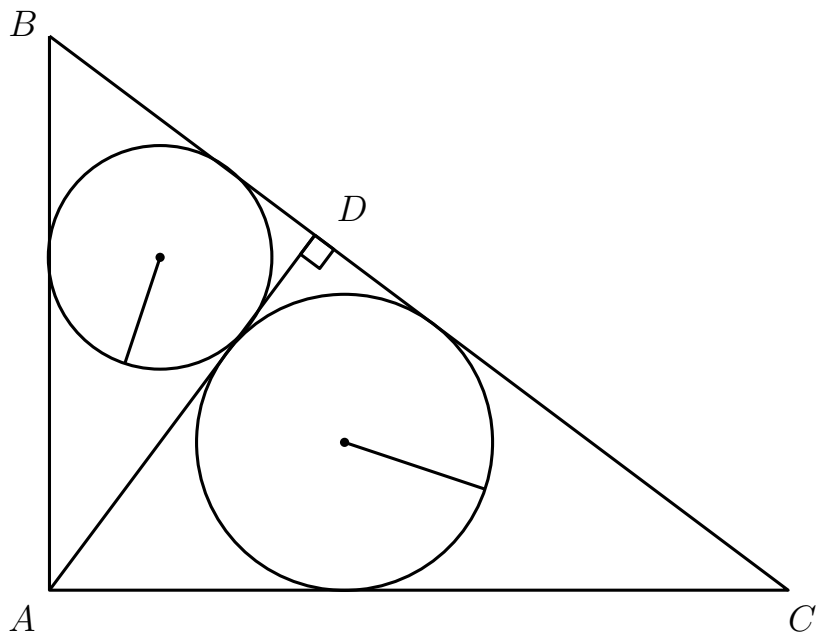
Suppose that real number x satisfies

$$\sqrt{49 - x^2} - \sqrt{25 - x^2} = 3.$$

What is the value of $\sqrt{49 - x^2} + \sqrt{25 - x^2}$?

Question 13

In right triangle ABC the side lengths are $AB = 15$, $AC = 20$, and $BC = 25$. Segment $\overline{AD}$ is the perpendicular to $\overline{BC}$. What is the sum of the radii of the circles inscribed in triangles ABD and ACD?



Question 14

We are given that integers x and y with $x > y > 0$ satisfy

$$x + y + xy = 80.$$

What is the value of x?

Question 15

What is the remainder when $f(x) = x^{2023} + 3x^2 + 14$ is divided by $x^2 - 1$?

Question 16

A *Pythagorean triple* is a triple of positive integers (a, b, c) for which $a^2 + b^2 = c^2$.

Notice that some Pythagorean triples such as $(3, 4, 5)$ and $(5, 12, 13)$ satisfy $c = b + 1$.

Find the Pythagorean triple (a, b, c) with $c = b + 1$ where $c < 1000$ is as large as possible.