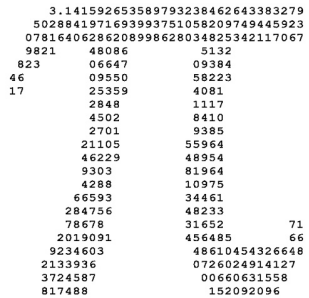
Carnegie Mellon University Qatar




# Pi Day Mathematics Competition

## Final Round 2019

# Question 1

While earlier attempts to calculate $\pi$ depended on polygonal approximations, more modern calculations use infinite series. One such series is the *Madhava-Leibniz* series:

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots = \frac{\pi}{4}.$$

What is the value of the following, related series

$$\frac{1}{1 \cdot 3} + \frac{1}{5 \cdot 7} + \frac{1}{9 \cdot 11} + \cdots ?$$

# Question 2

What is the sum of all positive integers $a$ which satisfy the condition $\frac{1}{15} < \frac{a}{10} < \frac{1}{3}$?

# Question 3

How many integers $x$ satisfy both of the two conditions

$$|3x + 8| = 3x + 8 \quad \text{and} \quad |2x - 5| = -2x + 5?$$

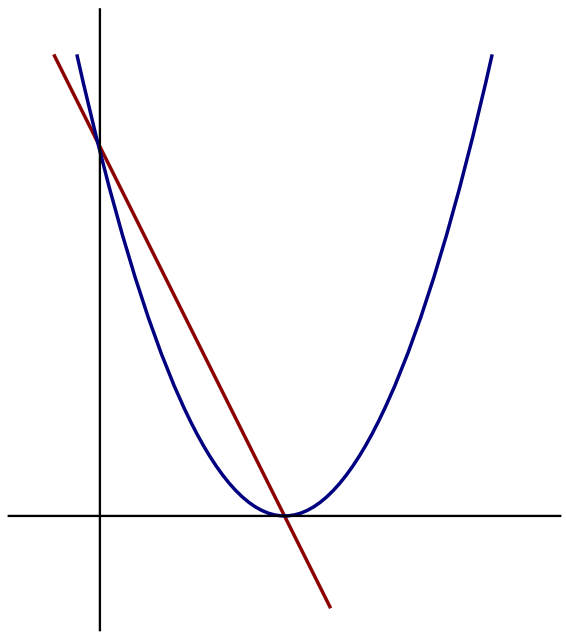
# Question 4

Suppose we are given that $4^x - 4^{x-1} = 24$.

What is the value of $x^{5/x}$?

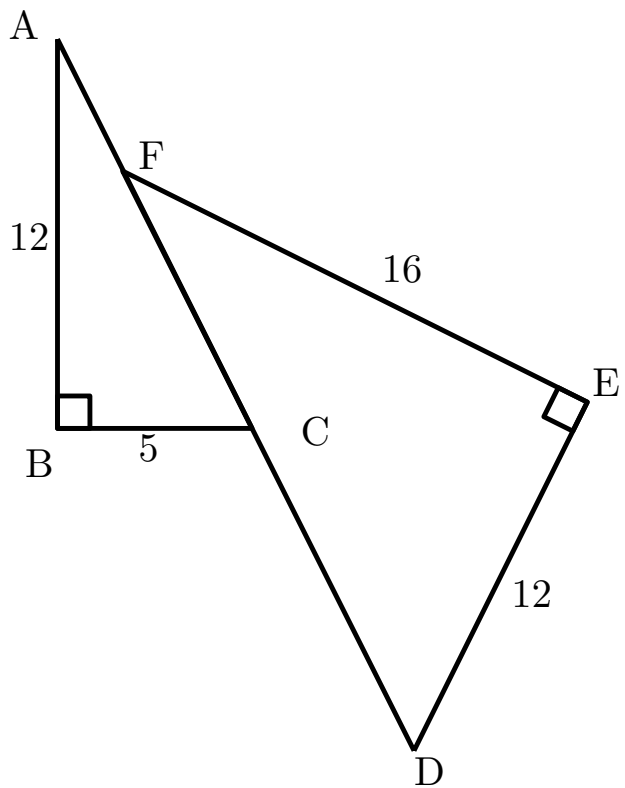
# Question 5

The parabola $y = ax^2 + bx + c$ and the line $y = -2x + 4$ have common $x$ and $y$ intercepts (as shown in the figure). What is the value of the sum $a + b + c$?



# Question 6

Let $ABC$ and $FED$ be right triangles. It is given that $|AB| = 12$ cm, $|BC| = 5$ cm, $|FE| = 16$ cm and $|DE| = 12$ cm. What is the difference $|DC| - |AF|$?



# Question 7

If $f(x-3) = (2n-1)x + 2m + 5$ is the identity function, what is the value of $n - m$?

# Question 8

Find an integer which is equal to the expression

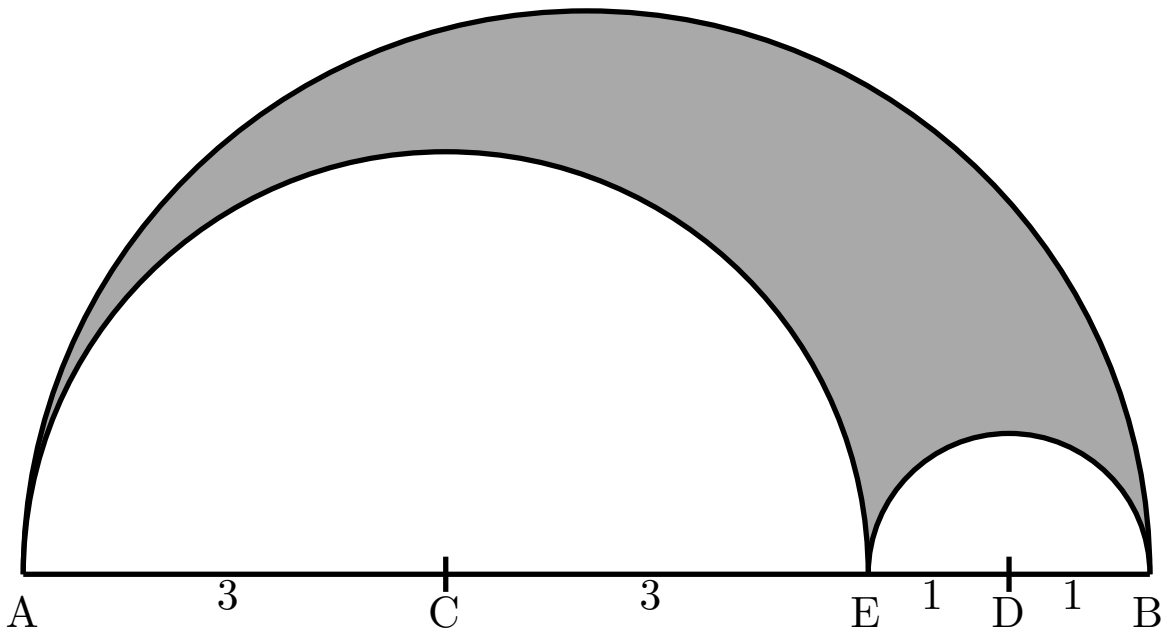
$$\left(\frac{0.003}{0.3} - \frac{0.0012}{0.12} + \frac{0.318}{31.8}\right)^{-1}.$$

# Question 9

What is the remainder when $3^1 + 3^2 + 3^3 + \cdots + 3^{2019}$ is divided by 10?

# Question 10

In the following figure, half-circles with diameters $|AB|$, $|AE|$, and $|EB|$ are given. If $|AC| = |CE| = 3$ cm and $|ED| = |DB| = 1$ cm then what is the area of the shaded (indicated) region?



# Question 11

Let $a, b, c, d, e$ be distinct integers such that

$$(7-a)(7-b)(7-c)(7-d)(7-e) = 75.$$

What is $a+b+c+d+e$?

# Question 12

We have three water pumps: A, B, and C.

It takes 6 hours for pump A, used alone, to fill a swimming pool.

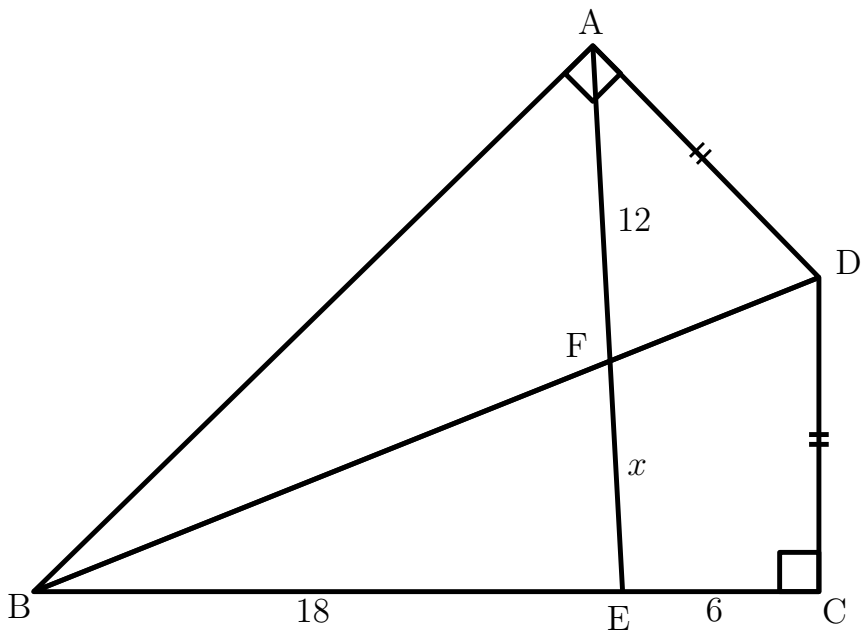
Pump B used alone takes 8 hours to fill the same pool.

Pump C is set up to drain/empty the pool. If the above pool is completely full, it takes pump C 12 hours to completely drain it.

Suppose pumps A and B are being used to fill the pool. When the pool is exactly half-full, pump C is turned on by accident. How long will it take to fill the remaining half with all pumps working?

# Question 13

Let $DAB$ and $BCD$ be right triangles. It is given that $|AD| = |DC|$, $|BE| = 18$ cm,$|EC| = 6$ cm and $|AF| = 12$ cm. What is the length of $|FE| = x$ in cm?



# Question 14

When the mean, median, and mode of the list of integers

$$11,\ 3,\ 5,\ 6,\ 3,\ 3,\ x$$

are arranged in increasing order, they form an arithmetic progression.

What is the sum of all possible values of $x$?

# Question 15

Two six-sided dice are fair in the sense that each face is equally likely to turn up. However, one of the dice has the 4 replaced by 3 and the other die has the 3 replaced by 4. When these dice are rolled, what is the probability that the sum is an odd number?

# Question 16

A *magic square* is a square grid such that the sum of entries in each row, column, and diagonal is equal. This common sum is called the *magic constant* of the magic square.

The $3 \times 3$ grid below is a magic square with some entries missing (and replaced by the letters $a, b, c, d, e, f$).

Determine its magic constant.

| | | |
|---|---|---|
| $15$ | $a$ | $b$ |
| $c$ | $d$ | $e$ |
| $20$ | $12$ | $f$ |