

Carnegie Mellon University Qatar

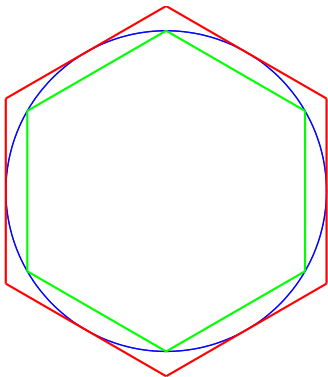
```
3.141592653589793238462643383279
5028841971693993751058209749445923
97816406286208998628034825342117067
9821. 48086 5332
823 24467 09284
46 39590 98223
17 23359 4081
2848 1117
4502 8410
2701 9585
21105 55964
48229 48954
8303 81364
4288 10975
60593 34401
284734 48233
78678 31452 71
2019091 456485 64
9234603 46010454326648
2128934 074626814127
3724587 00680431558
817488 152092086
```

Pi Day Mathematics Competition

Final Round 2018

Question 1

In the figure below, one regular hexagon is inscribed in a circle and another regular hexagon is circumscribed about the same circle. What is the ratio of the area of the smaller hexagon to the area of the larger hexagon?



Question 2

Let a and b be positive integers. If $\frac{1}{4}a + b = 8$, what is the largest value that a could take?

Question 3

What is the last digit in the decimal representation of 43^{2018} ?

Question 4

Fadhel has a fixed monthly salary and loves to save money each month. He puts $\frac{1}{24}$ of his monthly salary in a safe deposit box and $\frac{1}{x}$ of his monthly salary in a cookie jar. If after 15 months the sum of the money in the safe deposit box and cookie jar (total money that he has saved in 15 months) equals his monthly salary, what is the value of x ?

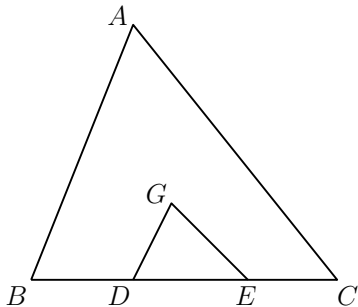
Question 5

Simplify the following sum:

$$\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots + \left(\frac{1}{2}\right)^{2018}$$

Question 6

Let G be the center of gravity (or centroid) of triangle ABC . If GD is parallel to AB , GE is parallel to AC , and the area of the triangle GDE is 8 cm^2 , what is the area of the triangle ABC ?



Question 7

If the polynomial $p(x) = x^4 + \frac{1}{2}x^3 + x^2 + ax$ can be divided without remainder by $x^2 + 1$, what is the value of a ?

Question 8

When a clock shows 4:15, what is the angle, in degrees, between the hour hand and the minute hand?

Question 9

Find the value of x where $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$

Question 10

Consider the geometric progression a, ar, ar^2, ar^3, \dots

If $-1 < r < 1$, it is known that the sum of the terms is $\frac{a}{1-r}$.

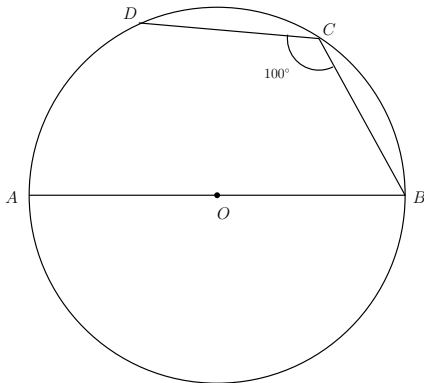
Find a simple expression for the sum of the squares of the terms in the above geometric progression.

Question 11

Suppose there are 10 students in a classroom. How many girls are there in the classroom if the number of different 2-person groups that can be selected among girls is equal to the number of boys?

Question 12

Let O be the center of the given circle. Suppose that $|DC| = |CB|$ and the measure of the angle BCD is 100° . What is the measure of the angle ABC ?



Question 13

Let α and β be the roots of the equation $x^2 - 4x - 5 = 0$.
Find the value of $\alpha^2 + 4\beta + 5$.

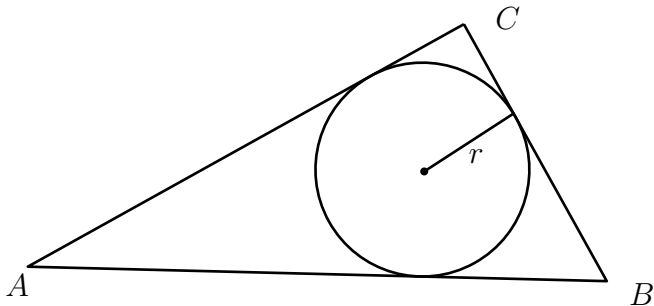
Question 14

There are 7 passengers in a train station. If these passengers are randomly distributed on 3 different train wagons, what is the probability that there will be exactly two passengers in the first wagon?

Question 15

In triangle ABC , the lengths of the sides are $AC = 24\text{cm}$, $BC = 10\text{cm}$, $AB = 26\text{cm}$.

What is the radius of the inscribed circle?



Question 16

For how many natural numbers n is the number $n^2 - 99$ a perfect square?

Tie-breaker 1

Let n be a positive integer and let $S(n)$ be the sum of the digits of n . (If $n = 474$, then $S(n) = S(474) = 4 + 7 + 4 = 15$).

What is the number of different positive integers n satisfying $n + S(n) = 2018$?

Tie-breaker 2

Let x_1 and x_2 be the roots of the equation $4x^2 - 5x - 1 = 0$.

Find the numerical value of the expression $\frac{1}{2 - x_1} + \frac{1}{2 - x_2}$.

Tie-breaker 3

The sequence 3, 1, 4, 1, 5, 9, 2, 6, ... is the sequence of digits of π in decimal form. What are the first five digits of π when it is expressed in binary?