

# Carnegie Mellon University Qatar

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07816406286208998628034825342117067  
9821 48086 5132  
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17 25359 4081  
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9234603 48610454326648  
2133936 0726024914127  
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First Annual Pi Day Mathematics Competition

Preliminary Round Question Booklet

2016

$\pi$  goes on and on,  
and  $e$  is just as cursed.  
I wonder, how does  $\pi$  begin  
When its digits are reversed?

- Martin Gardner

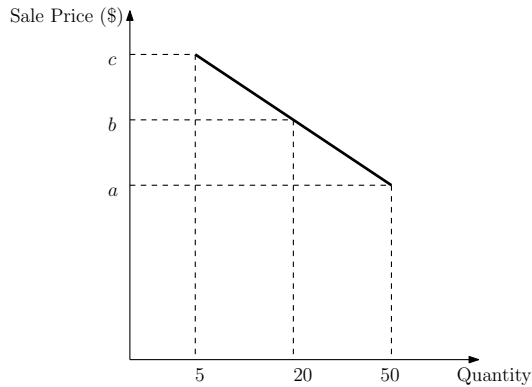


## Pi Day Mathematics Competition

### Rules of the Competition

- This test consists of 60 multiple choice questions. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
- Mark your answer to each problem on the provided answer sheet. For each question, blacken the circle corresponding to at most once answer choice. Completely erase errors and any stray marks. Only answers properly marked on the answer sheet will be graded.
- SCORING: You will receive one point for each correct answer. You receive 0 points for incorrect answers and for questions left unanswered.
- Contestants may **not** consult textbooks, notes, other people (apart from teammates), electronic devices (including calculators, mobile phones, etc.), or any other resources during the test.
- Figures are not necessarily drawn to scale.
- Before beginning the test, please make sure to write the name of your school, your team's name (if you have one) and the names of all members of the team on the answer sheet.
- You have **2 hours** to complete the questions.
- At the end of the 2 hours, each team should submit one answer sheet.

1. In general, the wholesale price of an item depends on the order size. The following graph shows this relation for a particular item. If  $c - a = 24$ , then what is  $c - b$ ?



- (a) 6                      (b) 8                      (c) 12                      (d) 14                      (e) 16
2. If  $x^2 < x$ , then what is the largest integer value of  $2x + 7$ ?
- (a) 6                      (b) 7                      (c) 8                      (d) 9                      (e) 10
3. If  $\frac{0.004x + 0.3}{0.007x + 0.05} = \frac{3}{4}$ , then  $x$  equals
- (a) 100                      (b) 120                      (c) 210                      (d) 121.8                      (e) 141.7
4. The sum to infinity of the terms of an infinite geometric progression is 9. The sum of the first two terms is 8. The first term of the progression is
- (a) 3                      (b) 4                      (c) 5                      (d) 6                      (e) 7
5. A woman has 100,000 QR to invest. If she invests 30,000 QR at 5% and 40,000 QR at 6%, at what rate is the remainder invested for the total interest to be 6,000 QR?
- (a) 6.5 %                      (b) 6.8 %                      (c) 7.0 %                      (d) 7.3 %                      (e) 7.5 %
6. The vertices of a parallelogram are located at points  $A = (3, 1)$ ,  $B = (5, 3)$ ,  $C = (2, 5)$ , and  $D = (a, b)$ . If  $BD$  is one of the diagonals of the parallelogram, find the length of this diagonal.
- (a) 1                      (b) 2                      (c) 3                      (d) 4                      (e) 5

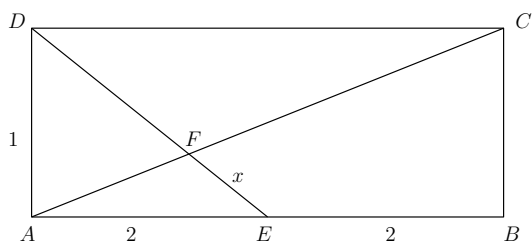
7. A number which when divided by 10 leaves a remainder of 9, when divided by 9 leaves a remainder of 8, when divided by 8 leaves a remainder of 7, etc., down to where, when divided by 2, it leaves a remainder of 1 is:

(a) 719                      (b) 1549                      (c) 2159                      (d) 2519                      (e) none

8. If  $|a - 2| + |b - 4| + |c - 6| = 0$ , then which of the following is the value of  $a + 2b + 3c$ ?

(a) 28                      (b) 12                      (c) 0                      (d) -12                      (e) -28

9. Refer to the figure given below. Let  $ABCD$  be a rectangle with  $|AD| = 1$  cm and  $|AE| = |EB| = 2$  cm. If  $|FE| = x$  cm, then find  $x$ .



(a)  $\frac{\sqrt{3}}{2}$                       (b)  $\frac{\sqrt{5}}{2}$                       (c)  $\frac{\sqrt{3}}{3}$                       (d)  $\frac{\sqrt{5}}{3}$                       (e)  $\frac{\sqrt{7}}{3}$

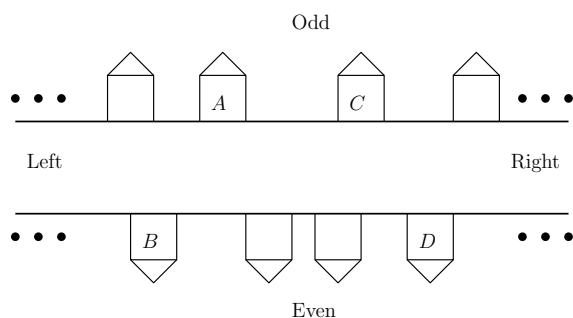
10. If we are given that  $\log_{20} 8 = .6941$ , which of the following are we not able to calculate without the use of a scientific calculator?

(a)  $\log_{20} 25$                       (b)  $\log_4 10$                       (c)  $\log_{20}(5/2)$                       (d)  $\log_5 9$                       (e)  $\log_{20} 16$

11. In a meeting, participants are offered tea, milk, or coffee. 7 of the participants do not drink tea, 6 of the participants do not drink milk, and 5 of the participants do not drink coffee. Moreover, 4 of the participants drink neither tea nor milk, 3 of the participants drink neither tea nor coffee, and 2 of the participants drink neither milk nor coffee. One person drinks none of the three beverages and no one drinks all three at the same time. How many persons are at the meeting?

(a) 10                      (b) 11                      (c) 12                      (d) 13                      (e) 14

12. In a street, house numbers are successive odd numbers on one side and successive even numbers on the other side. If the numbers are increasing from left to right, and  $A, B, C, D$  are the house numbers of the indicated houses with  $A - B = 15$ , what is  $C - D$ ?



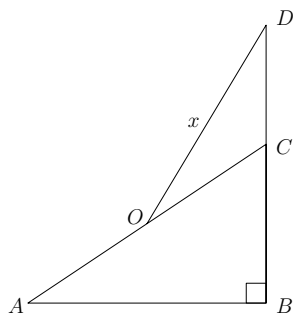
- (a) 9                      (b) 11                      (c) 13                      (d) 15                      (e) 17
13. Let  $f(x) = \frac{2x + u}{x + 1}$  and  $(f \circ f)(x) = \frac{x - 9}{3x - 2}$ , where  $\circ$  denotes composition of functions. What is  $u$ ?
- (a) -3                      (b) -2                      (c) -1                      (d) 0                      (e) 1
14. If each number in a set of twenty numbers is increased by 100, the arithmetic mean (average) of the numbers
- (a) remains the same                      (d) increases by 10  
 (b) increases by 20                      (e) increases by 2  
 (c) increases by 100
15. When a clock shows 2:15, the angle between the hour hand and the minute hand is:
- (a)  $15^\circ$                       (b)  $20^\circ$                       (c)  $22.5^\circ$                       (d)  $25^\circ$                       (e)  $30^\circ$
16. The equation  $x^4 + 4x^3 + 20x + 7 = 0$  has
- (a) no negative real roots                      (d) 1 positive and 1 negative real roots  
 (b) no positive real roots                      (e) 2 positive and 2 negative real roots  
 (c) no real roots

17. Mr. A owns a house worth 100,000 QR. He sells it to Mr. B at a 10% profit based on the worth of the house. Mr. B sells the house back to Mr. A at a 10% loss. Then,
- (a) A makes 10,000 QR on the deal                      (d) A loses 9,000 QR on the deal  
(b) A loses 10,000 QR on the deal                      (e) A comes out even  
(c) A makes 11,000 QR on the deal
18. A flower shop has some number of roses for Valentine's day. If roses are grouped in fives, then there are 2 extra roses. If roses are grouped in sevens, then there are 3 extra roses. What is the minimum possible number of roses that the flower shop may have?
- (a) 12                      (b) 17                      (c) 27                      (d) 37                      (e) 38
19. Let  $f(x) = mx + n$ . Assume that  $f^{-1}(3) = 4$  and  $f^{-1}(2) = 5$ , where  $f^{-1}(x)$  is the inverse of  $f(x)$ . What is the product of  $m$  and  $n$ ?
- (a) -7                      (b) -6                      (c) -5                      (d) 3                      (e) 6
20. The equation  $\sqrt{x + \sqrt{x + \sqrt{x + \dots}}} = 2$  is satisfied when  $x$  is
- (a) 1                      (b)  $\sqrt{2}$                       (c) 2                      (d)  $\sqrt[3]{2}$                       (e)  $\sqrt[4]{2}$
21. The product  $\log_b a \cdot \log_a b$  is equal to:
- (a) 1                      (b)  $a$                       (c)  $b$                       (d)  $ab$                       (e)  $\log_2 ab$
22. Given 12 points in a plane no three of which are collinear, the number of lines they determine is
- (a) 55                      (b) 66                      (c) 84                      (d) 100                      (e) 144

23. A point is selected at random inside an equilateral triangle. From this point, perpendiculars are drawn to each side of the triangle. The sum of these perpendiculars is
- (a) half the perimeter of the triangle
  - (b) equal to the length of a side of the triangle
  - (c) equal to the altitude of the triangle
  - (d) greatest when the point is the median
  - (e) least when the point is the median
24. The sum of the 3 digit number ABC and the two digit number AB is 392. What is the sum  $A + B + C$ ?
- (a) 7
  - (b) 9
  - (c) 11
  - (d) 15
  - (e) 19
25. A bag contains 10 white marbles and 10 black marbles. A person randomly selects a marble from the bag, places it on a table, and continues doing this until the bag is empty. What is the probability that the 3rd marble taken out is white?
- (a)  $\frac{1}{4}$
  - (b)  $\frac{1}{3}$
  - (c)  $\frac{1}{2}$
  - (d)  $\frac{2}{3}$
  - (e)  $\frac{3}{4}$
26. If  $\sqrt{x + \sqrt{x}} + \sqrt{x - \sqrt{x}} = 2$ , then  $x$  is
- (a)  $\frac{1}{3}$
  - (b)  $\frac{4}{3}$
  - (c) 1
  - (d) 2
  - (e)  $\sqrt{2}$
27. If  $x^{2016} - 2$  is divided by  $x + 1$ , the remainder is
- (a) -2
  - (b) -1
  - (c) 0
  - (d) 1
  - (e) 2



28. An isosceles triangle is a triangle with (at least) two equal sides. Let  $ABC$  be an isosceles right triangle. If  $|BD| = |AC| = 2$  cm,  $|OA| = |OC|$  and  $|OD| = x$  cm, then  $x$  equals



- (a)  $\sqrt{5 - 2\sqrt{2}}$     (b)  $\sqrt{5 - \sqrt{3}}$     (c)  $\sqrt{4 - 2\sqrt{2}}$     (d)  $\sqrt{4 - \sqrt{2}}$     (e)  $\sqrt{3 - \sqrt{2}}$
29. Let  $x = (2^3)^4$ ,  $y = 2^{(3^4)}$ , and  $z = (2^{12})^3$ . Which of the following is the correct ordering of  $x, y$ , and  $z$ ?
- (a)  $z < x < y$     (b)  $z < y < x$     (c)  $y < x < z$     (d)  $x < y < z$     (e)  $x < z < y$
30. If  $x^2 + 1$  is a factor of  $f(x) = x^4 + \frac{1}{2}x^3 + x^2 + ax$ , then  $a$  equals
- (a) 1    (b)  $\frac{1}{2}$     (c)  $\frac{1}{3}$     (d)  $-\frac{1}{3}$     (e) -1
31. It is given that one root of the quadratic equation  $x^2 + ax + b = 0$  is 3 and one root of the equation  $x^2 + cx + d = 0$  is -5. If the other roots of the equations are equal, find the value of  $a - c$ .
- (a) -8    (b) -6    (c) -3    (d) -2    (e) -1
32. Find the sum of the smallest and largest integer values of the expression  $\frac{30}{|2x + 6| + |x + 8|}$ .
- (a) 3    (b) 6    (c) 7    (d) 8    (e) 9
33. The difference of the roots of  $x^2 - 10x - 22$  is
- (a)  $2\sqrt{47}$     (b) 10    (c)  $\sqrt{47}$     (d)  $2\sqrt{10}$     (e)  $\sqrt{10}$

34. The digit "1" is used 689 times in the page numbers of a book. How many pages does the book have?

- (a) 1024                      (b) 1124                      (c) 1134                      (d) 1224                      (e) 1234

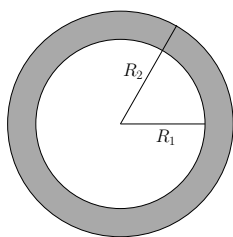
35. What is the arithmetic mean of  $\sqrt{6 - 2\sqrt{5}}$  and  $\sqrt{6 + 2\sqrt{5}}$  ?

- (a) 6                      (b) 12                      (c)  $\sqrt{5}$                       (d)  $\sqrt{6}$                       (e)  $6 + \sqrt{6}$

36. A farmer has 6 different size containers: 5, 9, 12, 23, and 45 liters. He has filled some of these containers with sunflower oil and some others with olive oil. He has one empty container and the total amount of the sunflower oil in the containers is 4 times the total amount of olive oil in the containers. What is the volume of the empty container?

- (a) 5                      (b) 9                      (c) 12                      (d) 15                      (e) 23

37. Refer to the figure given below. If  $R_1 + R_2 = 6$  cm and  $R_2 - R_1 = k$  cm, find the area between these two concentric circles.



- (a)  $3\pi k$  cm<sup>2</sup>                      (b)  $4\pi k$  cm<sup>2</sup>                      (c)  $6\pi k$  cm<sup>2</sup>                      (d)  $8\pi k$  cm<sup>2</sup>                      (e)  $9\pi k$  cm<sup>2</sup>

38. Find all real numbers for which  $\frac{(x^2 - 2)(x^2 + 4)}{(x^2 - 4)} < 0$ .

- (a)  $(-2, -\sqrt{2}) \cup (\sqrt{2}, 2)$                       (c)  $(-\infty, -\sqrt{2}) \cup (2, \infty)$                       (e)  $[-\sqrt{2}, 2]$   
(b)  $(-2, 0) \cup (\sqrt{2}, 2)$                       (d)  $(-\sqrt{2}, 2)$

39. The bottom, side, and front areas of a rectangular box are known. The product of these areas equals:

- (a) the volume of the box                      (d) the square of the volume  
(b) the square root of the volume                      (e) the cube of the volume  
(c) twice the volume

40. Fadhel looked at the wall clock when he started reading his newspaper. When he checked the clock again after he finished reading, he noticed that the hour and minute hands have switched positions. For how long did he read the newspaper?

(a) 40 min      (b)  $50\frac{5}{13}$  min      (c)  $54\frac{5}{13}$  min      (d)  $55\frac{5}{13}$  min      (e) 60 min

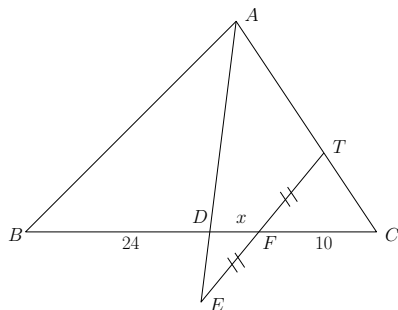
41. Maha just got a book from the library. She realizes that if she starts reading it today and if every day she reads five pages more than she read the previous day, she could read the entire book in 6 days. If she reads  $\frac{1}{3}$  of the book in three days using this rule, how many pages are there in the book?

(a) 126      (b) 129      (c) 132      (d) 134      (e) 135

42. In a geometric progression whose terms are all positive, any term is equal to the sum of the next two terms. Then the common ratio is:

(a)  $\frac{\sqrt{5}-1}{2}$       (b)  $\frac{\sqrt{5}+1}{2}$       (c)  $\frac{2-\sqrt{5}}{2}$       (d)  $\frac{2+\sqrt{5}}{2}$       (e)  $\frac{3-\sqrt{5}}{2}$

43. Refer to the figure given below. Note that  $|BD| = 24$  cm,  $|FC| = 10$  cm,  $|EF| = |FT|$ , and  $|DF| = x$  cm. If the line segments  $AB$  and  $TE$  are parallel, then  $x$  is



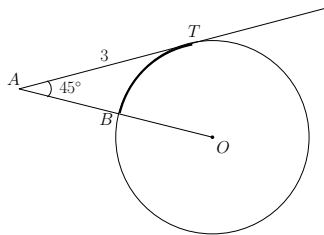
(a) 4      (b) 6      (c) 8      (d) 10      (e) 12

44. Let  $a, b$ , and  $c$  be positive real numbers. If  $\frac{a+b}{c} < \frac{a}{c} + 1$ , then which of the following inequalities is absolutely correct

(a)  $c < b$       (b)  $b < c$       (c)  $a < b$       (d)  $b < a$       (e)  $a < c$

45. Let  $\log_{10} 2 = a$  and  $\log_{10} 3 = b$ . Find the value of  $\log_{10} 72$  in terms of  $a$  and  $b$ .
- (a)  $2b - 3a$       (b)  $3a - b$       (c)  $3a - 2b$       (d)  $3a + 2b$       (e)  $2a + 3b$
46. If the radius of a circle is increased 100%, the area of the circle is increased
- (a) 50%      (b) 100%      (c) 200%      (d) 300%      (e) 400%
47. The sum to infinity of  $\frac{1}{4} + \frac{3}{4^2} + \frac{1}{4^3} + \frac{3}{4^4} + \cdots$  is
- (a)  $\frac{1}{5}$       (b)  $\frac{4}{3}$       (c)  $\frac{7}{3}$       (d)  $\frac{4}{15}$       (e)  $\frac{7}{15}$
48. Which of the following is the simplest expression equal to  $1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{a}}}$ .
- (a)  $1 + a$       (b)  $1 - a$       (c)  $-a$       (d)  $a$       (e)  $a - 1$
49. A six-sided die is loaded in a way that each even face is three times as likely as each odd face. All even faces are equally likely and all odd faces are equally likely. What is the probability that the outcome of a die roll will be a face showing 3 or less?
- (a)  $\frac{5}{12}$       (b)  $\frac{5}{13}$       (c)  $\frac{5}{14}$       (d)  $\frac{5}{15}$       (e)  $\frac{5}{16}$
50. Let  $x = \sin 85^\circ$ ,  $y = \tan 175^\circ$ ,  $z = \cos 260^\circ$ , and  $w = \cot 275^\circ$ . What is the correct sign of the given trigonometric quantities  $x, y, z$ , and  $w$ ?
- (a)  $+, -, +, -$       (b)  $-, -, -, +$       (c)  $+, -, -, +$       (d)  $-, -, -, -$       (e)  $+, -, -, -$
51. Two equal circles in the same plane cannot have the following number of common tangent lines:
- (a) 1      (b) 2      (c) 3      (d) 4
- (e) all the above are possible values for the number of common tangents.

52. Refer to the figure given below. Let  $O$  be the center of the given circle and let  $AT$  be a line segment tangent to the circle at point  $T$ . If  $|AT| = 3$  cm and  $m(\angle OAT) = 45^\circ$ , find the length of the circular arc  $BT$  in centimeters.

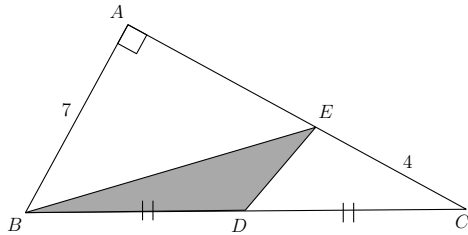


- (a)  $\frac{\pi}{2}$       (b)  $\frac{2\pi}{3}$       (c)  $\frac{3\pi}{4}$       (d)  $\frac{4\pi}{5}$       (e)  $\frac{5\pi}{6}$
53. Let  $\alpha \in (0^\circ, 90^\circ)$  and  $\sin \alpha = \frac{\sqrt{3} \sin 5 \cos 7 + \sqrt{3} \cos 5 \sin 7}{4 \cos 84 \cos 6}$ . What is  $\alpha$  in degrees?
- (a) 12      (b) 15      (c) 18      (d) 30      (e) 60
54. Ages of two sisters are proportional to 6 and 8 now. After 6 years, their ages will be proportional to 4 and 5 respectively. What is the age of the older sister now?
- (a) 26      (b) 24      (c) 20      (d) 18      (e) 16
55. Let  $A = \{a, b, c, d, e\}$ . Find the number of 3-element subsets that contain  $d$ .
- (a) 4      (b) 5      (c) 6      (d) 7      (e) 8
56. Alice and Bob take turns flipping a fair coin. The first person to get heads wins. If the game starts with Alice, what is the probability that Bob wins the game?
- (a)  $\frac{1}{4}$       (b)  $\frac{1}{3}$       (c)  $\frac{1}{2}$       (d)  $\frac{2}{3}$       (e)  $\frac{3}{4}$
57. Which of the following numbers is a rational number?
- (a)  $\sqrt{2} + 1$       (b)  $2\sqrt{2} - 1$       (c)  $\frac{1}{\sqrt{2}}$       (d)  $\frac{\sqrt{2}}{1 + \sqrt{2}}$       (e)  $\frac{2\sqrt{2} - 2}{3\sqrt{2} - 3}$

58. What is the smallest number divisible by 10 and such that the factorial of this number is divisible by  $10^{12}$ ?

- (a) 40                      (b) 50                      (c) 60                      (d) 80                      (e) 100

59. Refer to the figure given below and find the area of the triangle  $EBD$ . Note that  $m(\angle BAC) = 90^\circ$  (right angle),  $|AB| = 7$  cm,  $|EC| = 4$  cm, and  $|BD| = |DC|$ .

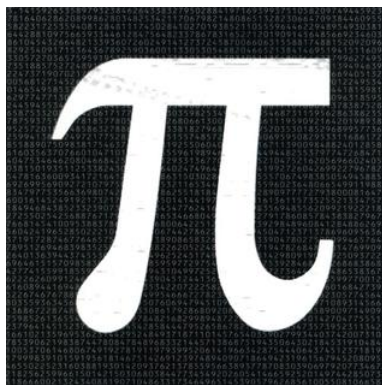


- (a) 3                      (b) 4                      (c) 7                      (d) 9                      (e) 11

60. Two classes of high school students take the same test. One class of 20 students had an average score of 85%; the other class of 30 students had an average score of 70%. What is the average score for all students in both classes?

- (a) 76%                      (b) 77%                      (c) 78%                      (d) 79%                      (e) 80%

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